Lecturered By Dr. H.K. Yadav Date - 16-05-2020
Deptt. of Mathematics Marwari College, DBG
B.Sc-part-I (Sub). Real Analysis

Q.1: ~~Deft~~ Define sequence and given an example.

Soln Sequence: → If, to each positive integer 1, 2, 3, ..., there corresponds a definite real number u_n, then the numbers $u_1, u_2, u_3, \ldots, u_n, \ldots$ are said to form a sequence.

A sequence of elements $u_1, u_2, \ldots, u_n, \ldots$ is denoted by (u_n) or by $\{u_n\}$.

A sequence is said to be finite or infinite according as the number of elements is finite or infinite.

For example: (i) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots$

(ii) $2, 4, 6, \ldots, 2n, \ldots$

Q. 2. Define convergent sequence and give an example.

Soln: Convergent Sequence: → The sequence $\{u_n\}$ is said to be convergent if the limit l of the sequence is a finite and definite number,

That is, having given a positive number ϵ, however small, there exists a positive integer m depending on ϵ such that $|u_n - l| < \epsilon$ for $n \geq m$, then $\{u_n\}$ is said to be convergent to the limit l, a definite number.

If $l = 0$, i.e. $u_n \to 0$ as $n \to \infty$, then $\{u_n\}$ is called a ~~Null~~ null sequence.

For example: $u_n = \frac{n}{n+1} \Rightarrow \lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{n}{n+1}$

$= \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 1$

And $u_n = a + \frac{(-1)^n b}{n}$

$\Rightarrow u_{2n} = a + \frac{(-1)^{2n} b}{2n} = a + \frac{b}{2n}$

$\therefore \lim_{n\to\infty} u_{2n} = \lim_{n\to\infty} \left(a + \frac{b}{2n}\right)$

$= a + \frac{b}{2} \lim_{n\to\infty} \frac{1}{n} = a + \frac{b}{2} \cdot 0$

$= a$

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Real Analysis

Q.2. State and prove Cauchy's general Principle of Convergence.

Soln: Statement:→ The necessary and sufficient condition for the convergence of any sequence $\{u_n\}$ is that corresponding to every arbitrary ϵ there exists a positive integer m such that

$$|u_{n+p} - u_n| < \epsilon,$$

for all values of $n \geq m$ and for all positive integral values of p.

proof: The condition is necessary

Let $u_n \to l$ as $n \to \infty$.

Then, $|u_n - l| < \frac{\epsilon}{2}$, when $n \geq m$

Since, $n+p > m$, therefore, $|u_{n+p} - l| < \frac{\epsilon}{2}$, where p is any positive integer.

$$\therefore |u_{n+p} - u_n| = |(u_{n+p} - l) - (u_n - l)|$$
$$\leq |u_{n+p} - l| + |u_n - l|$$
$$< \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$$

when $n \geq m$ and p is any positive integer.

The condition is sufficient

Let $|u_{n+p} - u_n| < \epsilon$, when $n \geq m$ and p is any +ve integer.

Then, $u_n - \epsilon < u_{n+p} < u_n + \epsilon$, when $n \geq m$ and p is any +ve integer.

Thus $\{u_{n+p}\}$ is bounded when $p \to \infty$.

If N and M be the lower and upper bounds respectively then, $N \geq u_n - \epsilon$ and $M \leq u_n + \epsilon$.

$$\therefore M - N \leq u_n + \epsilon - u_n + \epsilon = 2\epsilon$$

Since, ϵ is an arbitrarily small positive number, therefore

$$M - N = 0 \text{ i.e. } M = N$$
$$\therefore M - \epsilon < u_{n+p} < M + \epsilon$$

i.e. $|u_{n+p} - M| < \epsilon$ i.e. $u_{n+p} \to M$ as $p \to \infty$.

Hence, $\{u_n\}$ converges to M.

proved